”Latent heat” of first-order varying pressure transitions

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Abstract

We consider the energy release associated with first-order transition by Gibbs construction and present such energy release as an accumulation of a series of tiny binding energy differences between over-compressed states and stable ones. Universal formulae for the energy release from one homogeneous phase to the other is given. We find the energy release per converted particle varies with number density. As an example, the deconfinement phase transition at supranuclear densities is discussed in detail. The mean energy release per converted baryon is of order 0.1MeV in RMF theory and MIT bag descriptions for hadronic matter and strange quark matter for a wider parameter region.

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1 Introduction

Glendenning[1] had realized the essentially different character of a first-order transition in simple system possessing a single conserved quantity and complex one having more than one conserved charge. One of the most remarkable features of a simple system is the constancy of the pressure during the transition from one homogeneous phase to the other. Such phase transition is the typical description of first-order one in textbook. The properties of the transition are quite different in complex. The pressure varies continuously with the proportion of two pure phases in equilibrium. Some quantities are obviously nonlinear functions in the proportion in the mixed phase.

As well-known, the occurrence of first-order transition will eventually accompany the release of energy. The energy is called latent heat related to the release of entropy in the example of gas-liquid transition. The phase transition at zero temperature showed the liberation of binding energy. We still use the concept of ”latent heat” for the release of energy associated with such the first-order phase transition.

The calculations of ”latent heat” from hadronic matter to strange quark matter were firstly made by Haensel and Zdunik[2] . They considered hadrons be absorbed into strange quark matter in accreting strange stars and had expected that the heat per absorbed neutron is in the range 10$\sim$30MeV. Generally the energy release for constant-pressure phase transition at zero temperature takes place nearby a transition point where

an over-compressed state, metastable, is probably the first to form and then go to a stable state[3]. However, the phase transition in a multicomponent system takes place in a varying pressure region. An appearance of over-compressed state at every pressure is possible. So we need to face up to a series of metastable states if the assumed system has a durative transfer from one pure phase to the other. Relatively speaking, the treatment of such energy release is a more difficult task than the constant-pressure case. We will focus on solving the problem in present paper.

Many phase transitions in nuclear matter are the expected ones having multicomponent mixture in chemistry, the nuclear gas-liquid transition relevant to accelerator experiments, where the neutron and proton number are conserved, pion, koan condensates and deconfinement of quarks in high density, where the conserved charges are baryon and electric. Typically, we will treat the problem on the converted energy taking the confined-deconfined phase transition as an example.

Ideally the two phase should be described selfconsistently in the same theory, but we cannot solve QCD through confined-deconfined phase transition based on our present knowledge. Hence we will use separate models for each phase. We describe the equation of state of hadronic matter in the framework of the relativistic mean-field theory and depict the deconfined quark matter in the MIT bag model. We will study the baryon number density dependence of energy per baryon for different phases, hadronic, quark and their mixed phase.

The plan of the paper is as follows. In Sec. II we state our views on "latent heat" and formulate the problem. In Sec.III we solve numerically the equations in the confined-deconfined phase transition for the given equations of state. Our discussion and conclusion will be made in Sec.IV.

2 Formulation of the problem

For superdense nuclear matter where a first-order phase transition would take place, the pressure p versus baryon number density could generally be described in Figure 1 if we follow the same philosophy and method used by Glendenning. Correspondingly, the number density dependence of the energy per baryon could also be revealed in the figure(low panel). The body experiences from low density phase, mixed phase to high density phase with the number density increase. We find the suppression of the increase binding energy in the mixed phase as compared with the low density phase when the body gradually compressed. The starting point S in the mixed phase is so conspicuous that the body may go beyond S in low density phase to an over-compressed state and then have a transition to a stable state in mixed phase. Analogous situation would appear at every point in mixed phase region. The process depicted by A, B and C should repeatedly occur through the duration when the body were transferred from S to F. We can consider that the transfer is nearly along the line m, denoting the equation of state, if the over-compressed states have very small deviations from the corresponding state in mixed phase region.

As well-known, the total energy and baryon number densities for the mixed phase,

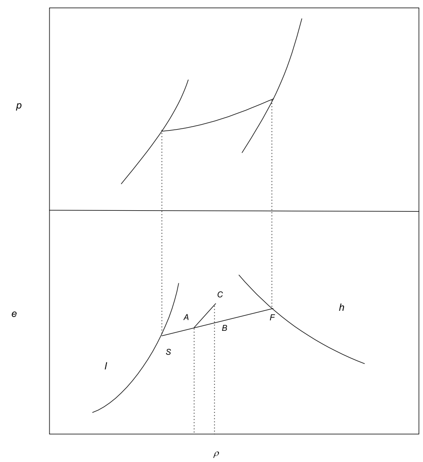

Figure 1: The schematic diagram for the equation of state having more than one conserved charge

given by Glendenning[1, 4], read

$$\epsilon = \chi\epsilon_h + (1-\chi)\epsilon_l, \tag{1}$$

$$\rho = \chi\rho_h + (1-\chi)\rho_l. \tag{2}$$

with the volume fraction

$$\chi = \frac{V_h}{V_l + V_h} \tag{3}$$

where V_l and V_h represent the volumes occupied by low density and high density phase respectively, ϵ_l and ϵ_h denote the corresponding energy densities, ρ_l and ρ_h, the baryon number densities.

We further introduce the baryon number fraction $\eta(= N_h/N)$ to rewrite the equations (1) and (2) for convenience of the following treatments,where N_h and N present high density phase and total baryon numbers.

Substitute $V_l = N_l/\rho_l, V_h = N_h/\rho_h$ into Eq(3), we get

$$\chi = \frac{\eta\rho_l}{\eta\rho_l + (1-\eta)\rho_h} = \eta\frac{\rho}{\rho_h} \tag{4}$$

$$1-\chi = \frac{(1-\eta)\rho_h}{\eta\rho_l + (1-\eta)\rho_h} = (1-\eta)\frac{\rho}{\rho_l} \tag{5}$$

Therefore Eqs(1) and (2) become

$$\epsilon = \frac{\eta\rho_l}{\eta\rho_l + (1-\eta)\rho_h}\epsilon_h + \frac{(1-\eta)\rho_h}{\eta\rho_l + (1-\eta)\rho_h}\epsilon_l \tag{6}$$

$$\rho = \frac{\rho_h \rho_l}{\eta\rho_l + (1-\eta)\rho_h} \tag{7}$$

The energy per baryon(binding energy) is expressed as

$$e = \frac{\epsilon}{\rho} = \eta e_h + (1-\eta)e_l \tag{8}$$

At zero temperature the energy e only depends on the density. The change in e arises from the density increases by means of compression and deconfinement.The energy e can therefore be assumed to be of the function of form $e(\eta(\rho), \rho)$. We find the derivative of e with respect to ρ, which reads

$$\frac{\delta e}{\delta \rho} = \left(\frac{\partial e}{\partial \rho}\right)_\eta + \frac{\partial e}{\partial \eta}\frac{\delta \eta}{\delta \rho}. \tag{9}$$

Furthermore

$$\delta e = e + \frac{\delta e}{\delta \rho} = e + \left(\frac{\partial e}{\partial \rho}\right)_\eta \delta\rho + \frac{\partial e}{\partial \eta}\delta\eta, \tag{10}$$

where

$$\left(\frac{\partial e}{\partial \rho}\right)_\eta = \eta\left(\frac{\partial e_h}{\partial \rho}\right)_\eta + (1-\eta)\left(\frac{\partial e_l}{\partial \rho}\right)_\eta. \tag{11}$$

Eq(10) shows the fact that the increase of binding energy in mixed phase amounts to the composition of two processes, the simple compressional change without phase transition and the compressional one due to the occurrence of phase transition. We define

$$-\delta q = \frac{\partial e}{\partial \eta}\delta\eta, \tag{12}$$

and hence substitute it into Eq(9), we immediately have

$$\delta q = \left[\left(\frac{\partial e}{\partial \rho}\right)_\eta - \frac{\delta e}{\delta \rho}\right]\delta\rho = \left[(e + \left(\frac{\partial e}{\partial \rho}\right)_\eta \delta\rho) - (e + \frac{\delta e}{\delta \rho}\delta\rho)\right] \tag{13}$$

In equation(11), η has been supposed invariable factor with increase density. $\left(\frac{\partial e}{\partial \rho}\right)_\eta$ indicates the the binding energy increase by a simple compression regardless of phase transition since no particle is converted to the other phase from one phase. The second term of the right-hand side in Eq(9) or Eq(12) is obviously associated with phase transition because it arises from the change of the baryon number fraction. We further change Eq(13) to reveal the physical meanings of quantity δq. From an initial state(see A in figure 1) to the adjacent stable state, the system suffering phase transition(see B), or the adjacent metastable one, the system experiencing simple compression(see C), through a equivalent density change, the binding energies for B and C can be calculated as $\frac{E_B}{N} = e + \frac{\delta e}{\delta \rho}\delta\rho$ and $\frac{E_C}{N} = e + \left(\frac{\delta e}{\delta \rho}\right)_\eta \delta\rho$. The δq is immediately the energy difference between the metastable state and the stable one, i.e.$\delta q = \frac{E_C}{N} - \frac{E_B}{N}$, where E_B, E_C represent total energies of the system for the two states. Evidently δq indicates the binding energy release, just the same mechanism as the physical situation in

[2], when the body change from the metastable state C into the stable state$B$. A very small change of density considered here is the only difference from Ref[2] and hence the energy release, δq, tends to zero. But the accumulation of a series of infinitesimal released energies is able to be nonvanishing because the system certainly experiences the finite change during real phase transition, which can be expressed by an definite integral.

Therefor, when we consider the assumed system with N baryon number from pure low density phase(S in figure 1) to pure high density one(F in the figure), the energy release can be expressed as

$$Q = N\bar{q} = \int_{\mathrm{S}}^{\mathrm{F}} N\delta q = N\int_{\mathrm{S}}^{\mathrm{F}} \left[\left(\frac{\partial e}{\partial \rho}\right)_{\eta} - \frac{\delta e}{\delta \rho}\right]\delta\rho, \tag{14}$$

where $\bar{q}$ is mean energy release per baryon. We can also calculate the energy release per converted baryon for different densities. The total energy release for given density ρ in mixed phase region equals $N\delta q$. Meanwhile, the converted baryon numbers should be δN_h at infinitesimal compression $\delta\rho$. Thus the energy release per converted baryon is calculated by

$$q(\rho) = \frac{N\delta q}{\delta N_{\mathrm{h}}} = \left(\frac{\frac{\delta q}{\delta \rho}}{\frac{\mathrm{d}\eta}{\mathrm{d}\rho}}\right) \tag{15}$$

The meanings of the formula will be displayed thoroughly when Eq(12) is substituted into Eq(15),

$$q(\rho) = \frac{\partial e}{\partial \eta} \tag{16}$$

Clearly the energy release per converted baryon is different for various baryon number density during phase transition in varying pressure. We turn back to the understandings of Eq(10). We easily find that the simple compression causes the binding energy increase but the phase transition leads to the reduction. The reduced energy is just about the energy release.

3 Application: The energy release in deconfinement phase transition

We will numerically solve the integral(12) together with equations(1)and(2) for the transition from hadronic matter to quark matter. The low density phase represents hadronic matter, called HP for short, and the high density one indicates quark matter, QP for short. We use the equations of state in relativistic mean-field theory(RMF) description for hadronic matter and the equations of state in MIT bag model for quark matter.

$$\begin{aligned} Ł = \sum_B \overline{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\tau\cdot\rho^\mu)\psi_B \\ +\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) + \sum_{\lambda=e,\mu}\overline{\psi}_\lambda(i\gamma_\mu\partial^\mu - m_\lambda)\psi_\lambda \end{aligned}$$

$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu-\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu}$$
$$+\frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu-\frac{1}{3}bm_n(g_{\sigma n}\sigma)^3-\frac{1}{4}c(g_{\sigma n}\sigma)^4. \tag{17}$$

where the spinor for the baryon species B is denoted by ψ_B, σ, ω, ρ represent meson fields. The energy density and pressure can be obtained in the familiar manner as

$$\begin{aligned}\epsilon_{HP} = \frac{1}{3}bm_n(g_\sigma\sigma)^3+\frac{1}{4}c(g_\sigma\sigma)^4+\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{2}m_\omega^2\omega_0^2+\frac{1}{2}m_\rho^2\rho_{03}^2 \\ +\sum_B\frac{2J_B+1}{2\pi^2}\int_0^{k_B}\sqrt{k^2+(m_B-g_{\sigma B}\sigma)^2}k^2dk \\ +\sum_\lambda\frac{1}{\pi^2}\int_0^{k_\lambda}\sqrt{k^2+m_\lambda^2}k^2dk.\end{aligned} \tag{18}$$

$$\begin{aligned}p_{HP} = -\frac{1}{3}bm_n(g_\sigma\sigma)^3-\frac{1}{4}c(g_\sigma\sigma)^4-\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{2}m_\omega^2\omega_0^2+\frac{1}{2}m_\rho^2\rho_{03}^2 \\ +\frac{1}{3}\sum_B\frac{2J_B+1}{2\pi^2}\int_0^{k_B}\frac{k^4}{\sqrt{k^2+(m_B-g_{\sigma B}\sigma)^2}}dk \\ +\frac{1}{3}\sum_\lambda\frac{1}{\pi^2}\int_0^{k_\lambda}\frac{k^4}{\sqrt{k^2+m_\lambda^2}}dk.\end{aligned} \tag{19}$$

where the coupling constants g_σ,g_ω,g_ρ,b and c can be determined by the nuclear saturation density ρ_0, the binding energy at saturation *B/A*, the symmetry energya_{sym}, the compression modulus*K* and the effective nucleon mass m^*. In MIT bag model, the energy density and pressure of quark matter at zero temperature are given by

$$\epsilon_{QP} = B+\sum_f\frac{3}{4\pi^2}\left[\mu_f(\mu_f^2-m_f^2)^{1/2}(\mu_f^2-\frac{1}{2}m_f^2)-\frac{1}{2}m_f^4\ln\left(\frac{\mu_f+(\mu_f^2-m_f^2)^{1/2}}{m_f}\right)\right]. \tag{20}$$

$$p_{QP} = -B+\sum_f\frac{1}{4\pi^2}\left[\mu_f(\mu_f^2-m_f^2)^{1/2}(\mu_f^2-\frac{5}{2}m_f^2)+\frac{3}{2}m_f^4\ln\left(\frac{\mu_f+(\mu_f^2-m_f^2)^{1/2}}{m_f}\right)\right]. \tag{21}$$

We calculate the mixed phase of hadronic and quark matter with Gibbs construction which first used by Glendenning[1]. For simplicity, we neglect Coulom and surface effects in MP. The Gibbs condition for mechanical and chemical equilibrium at zero temperature in MP is written as

$$p_{\mathrm{PH}}(\mu_n,\mu_e) = p_{\mathrm{QP}}(\mu_n,\mu_e) \tag{22}$$

We will obtain Eqs(1) and (2) by applying the condition (20) together with Eqs(16)$\sim$(19) and imposing the condition of global charge neutrality in MP

$$\chi\rho_{\mathrm{QP}}^c+(1-\chi)\rho_{\mathrm{HP}}^c = 0, \tag{23}$$

Table 1: Nucleon-meson coupling constants

Name	$(\frac{g_\sigma}{m_\sigma})^2(\mathrm{fm}^2)$	$(\frac{g_\omega}{m_\omega})^2(\mathrm{fm}^2)$	$(\frac{g_\rho}{m_\rho})^2(\mathrm{fm}^2)$	$100b$	$100c$	Ref
RMF1	11.79	7.149	4.411	0.2947	-0.1070	[4]
RMF2	8.492	4.356	5.025	0.2084	2.780	[5]
RMF3	10.339	4.820	4.791	1.1078	-0.9751	[4]

where ρ^c_{QP} and ρ^c_{HP} respectively denote negative charged density of quark matter and positive one of hadronic matter. When χ=0 and 1, the global charge neutrality spontaneously come back local charge neutralities for pure hadronic matter and pure quark matter. The equilibrium chemical potentials can be determined in MP from Eq(20) while χ can be solved from Eq(21).

We choose the representative parameters for soft, moderate and stiff hadronic matter equations of state listed in table 1. The bag constant is taken as $B^{1/4}$=170MeV, 180MeV and 190MeV. We construct a series of the equations of state with MP denoted by RMFn+$B^{1/4}$(n=1,2,3) under those considerations. As an example, RMF1+180 is depicted in figure 2.

We calculates $\frac{\delta q}{\delta \rho}$ and $\bar{q}$ for the different equations of state. The results are showed in figure 3 and 4 . Although the uncertainties of the equations of state have effect on the results, they are of order 0.1MeV under our considerations.

We also utilize equation (15) to calculate $q(\rho)$ shown in figure 5 and 6, corresponding to figure 3 and 4. Obviously, $q(\rho)$ changes around $\bar{q}$ in the wide regions. The data for several cases, RMF1+180, RMF2+180 and RFM3+180, are listed in tables 2$\sim$4.

4 Conclusion and discussion

We put forward a method to calculate the energy release in the first-order phase transition that have varying pressure found by Glendenning. We formulate the release of energy per baryon from one homogenous phase to the other. We find the dependence of energy release per converted baryon on density.

We investigate the case of deconfinement phase transition using RMF description for hadronic matter and MIT bag description for quark matter. We find that the mean energy release per converted baryon is of order 0.1MeV, much smaller than that of the transition considered by Haensel and Zdunik, tens of MeV[2]. We indeed see the difference of energy release if a hadron is converted into quarks at different pressure(density) points.

From the analysis and the performance of mathematical calculations, we see that the binding energy in a hadron is probably liberated during transition process at zero temperature. Clearly the degree of freedom for a baryon number changes in deconfinement phase transition. Our calculations also show that the deconfinement process in varying pressure can be regard as an accumulation of innumerable constant-pressure phase transitions.

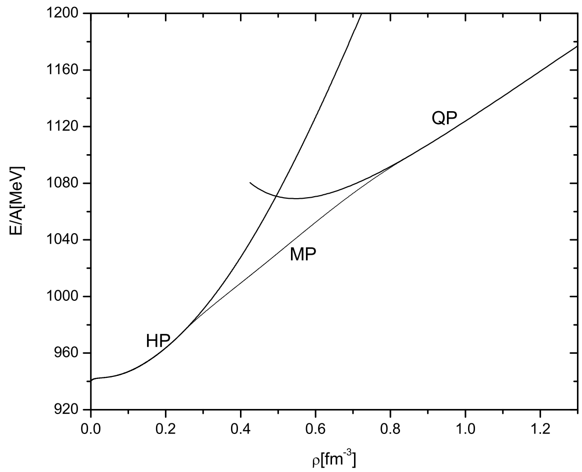

Figure 2: Energy per baryon versus baryon number density for RMF1+180

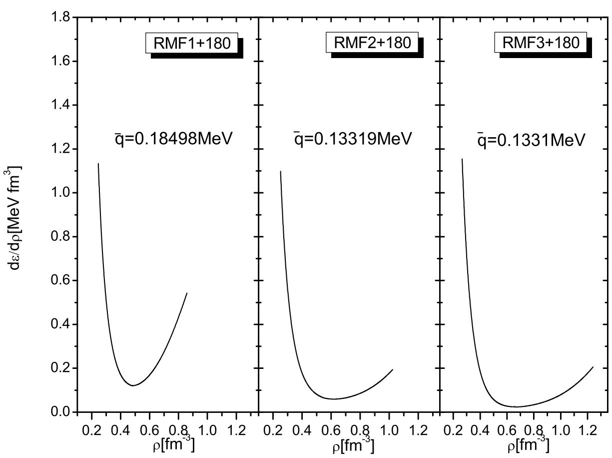

Figure 3: The $\frac{\delta q}{\delta \rho}$ versus ρ for soft, moderate and stiff hadronic matter equation of state. The bag constant is given $B^{1/4} = 180\text{MeV}$

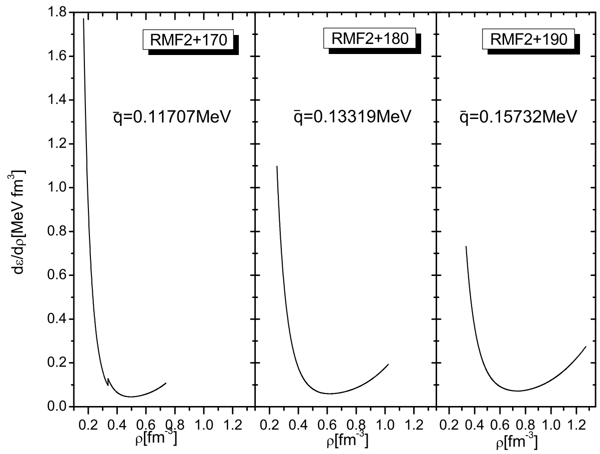

Figure 4: The $\frac{\delta q}{\delta \rho}$ versus ρ for different bag constant models. The hadronic matter equation of state is given as moderate soft one.

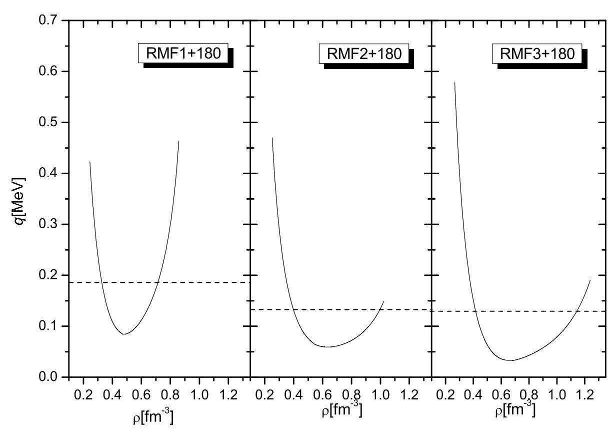

Figure 5: The baryon number density dependence of releasing energy per a converted baryon for soft, moderate and stiff hadronic matter equation of state. The horizontal lines represent the mean values.

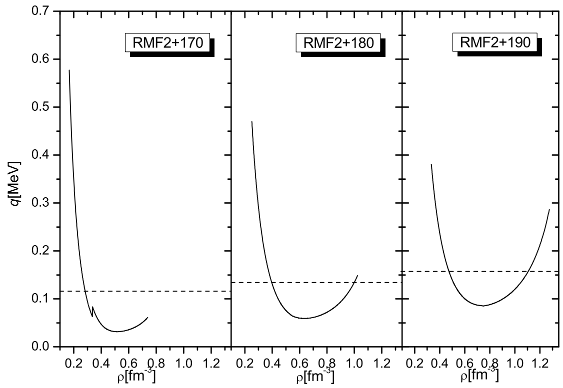

Figure 6: The baryon number density dependence of releasing energy per a converted baryon for different bag constants. The horizontal lines represent the mean values.

Our discussion may be relevant to accelerator experiments and many astrophysical problems. Especially, many transitions with two conserved charges could occur in neutron stars, such as nuclei compositional transition in the crust[6], meson condensates[7], hyperon productions and superfluid transitions in the interior[8]. The consideration in this work may be applied to them. The energy release due to such first-order transitions would significantly influence the evolution of neutron stars[6, 9, 10].

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Table 2: The calculations of $q(\rho)$corresponding to the equation of state of RMF1+180

$\rho(fm^{-3})$	$P(MeVfm^{-3})$	η	$q(\rho)(MeV)$
0.24562	15.99590	0.00252	0.419183681
0.28521	18.92140	0.10374	0.275781000
0.32505	22.89477	0.18689	0.189162009
0.36542	28.14901	0.25904	0.136300761
0.40539	34.62885	0.32302	0.105876989
0.42517	38.28373	0.35297	0.096513344
0.44539	42.28858	0.38282	0.089926622
0.46579	46.57121	0.41241	0.085636798
0.48530	50.85101	0.44036	0.084306666
0.49537	53.10652	0.45483	0.084855744
0.50556	55.40860	0.46950	0.085843691
0.51543	57.65472	0.48379	0.087173365
0.52501	59.83958	0.49772	0.088797472
0.53523	62.17201	0.51266	0.090895052
0.54519	64.44081	0.52732	0.093310771
0.55540	66.75197	0.54243	0.096084463
0.56539	68.99347	0.55730	0.099108422
0.57515	71.16039	0.57192	0.102399167
0.58520	73.36496	0.58709	0.106045033
0.59505	75.48930	0.60201	0.109986619
0.60524	77.64971	0.61755	0.114346182
0.61583	79.84750	0.63378	0.119247950
0.63516	83.72478	0.66360	0.129212059
0.65549	87.59585	0.69515	0.141124493
0.67551	91.18872	0.72634	0.154501478
0.69505	94.47733	0.75679	0.169333315
0.71571	97.72125	0.78892	0.187330950
0.73563	100.6232	0.8197	0.207391595
0.75550	103.3067	0.85015	0.230686953
0.77510	105.7551	0.87984	0.257760827
0.79554	108.1105	0.91037	0.291723249
0.81526	110.2041	0.93934	0.332089997
0.83551	112.1845	0.96855	0.384646739
0.85620	114.0450	0.99778	0.447912000

Table 3: The calculations of $q(\rho)$corresponding to the equation of state of RMF2+180

$\rho(fm^{-3})$	$P(MeVfm^{-3})$	η	$q(\rho)(MeV)$
0.25134	15.73143	0.00151	0.469751750
0.29175	18.45115	0.09636	0.313446165
0.33167	21.94685	0.17143	0.219698869
0.37161	26.38217	0.23424	0.160179216
0.41056	31.70503	0.28737	0.122075922
0.44546	37.35738	0.33025	0.098760648
0.45562	39.15959	0.34209	0.093479673
0.46561	41.00209	0.35350	0.088757745
0.47594	42.97941	0.36510	0.084349887
0.48566	44.90266	0.37581	0.080640911
0.49573	46.96361	0.38676	0.077251319
0.50570	49.06666	0.39744	0.074219457
0.51558	51.21138	0.40790	0.071506976
0.52583	53.50250	0.41863	0.069013661
0.53555	55.73141	0.42870	0.066841209
0.54566	58.11011	0.43906	0.064591642
0.55527	60.42206	0.44884	0.063371717
0.56576	63.00044	0.45949	0.062230485
0.57578	65.51184	0.46965	0.061360053
0.58580	68.06732	0.47980	0.060609410
0.59582	70.66685	0.48995	0.060077471
0.60540	73.18941	0.49966	0.059635930
0.61546	75.87528	0.50989	0.059324310
0.62555	78.60550	0.52018	0.059157773
0.63570	81.38022	0.53056	0.059148265
0.64544	84.07060	0.54056	0.059176934
0.65524	86.80200	0.55068	0.059341975
0.67079	91.17776	0.56685	0.059817394
0.69090	96.89657	0.58802	0.060746128
0.71096	102.6439	0.60947	0.062017483
0.73053	108.2654	0.63074	0.063615371
0.75066	114.0416	0.65302	0.065581225
0.77091	119.8231	0.67590	0.067979420
0.81089	131.0503	0.72245	0.073798485
0.85070	141.8468	0.77076	0.081456043
0.89050	152.1159	0.82099	0.091154274
0.93036	161.7624	0.87312	0.103677438
0.97023	170.6913	0.92685	0.119589566
1.02250	181.2445	0.99918	0.148039000

Table 4: The calculations of $q(\rho)$corresponding to the equation of state of RMF3+180

$\rho(fm^{-3})$	$P(MeVfm^{-3})$	η	$q(\rho)(MeV)$
0.26621	15.38213	0.00033	0.578528090
0.30013	17.25338	0.07188	0.401621596
0.35009	20.96879	0.15573	0.242218167
0.39063	25.04845	0.21094	0.164227120
0.41041	27.45226	0.23484	0.136629769
0.43008	30.14255	0.25704	0.114235389
0.44629	32.59680	0.27429	0.098882871
0.46224	35.23192	0.29047	0.086101857
0.47848	38.14854	0.30624	0.075092011
0.49401	41.16360	0.32073	0.066246291
0.50990	44.47685	0.33500	0.058676162
0.52612	48.10325	0.34909	0.052223703
0.54224	51.94964	0.36267	0.047056820
0.56641	58.15899	0.38235	0.041151896
0.58231	62.52432	0.39493	0.038316741
0.60639	69.53827	0.41355	0.035288569
0.62200	74.32911	0.42539	0.034077900
0.64609	82.06479	0.44342	0.033020453
0.66213	87.42851	0.45532	0.032808586
0.68627	95.78867	0.47311	0.032873272
0.70224	101.4816	0.48487	0.033734009
0.72612	110.1944	0.50259	0.035371294
0.74231	116.2149	0.51474	0.036785477
0.76631	125.2663	0.53299	0.039089761
0.78216	131.3051	0.54525	0.040836874
0.80619	140.5170	0.56417	0.043644712
0.82229	146.7050	0.57712	0.045721299
0.86230	162.0314	0.61029	0.051421958
0.90207	177.0247	0.64485	0.058017367
0.94236	191.7758	0.68158	0.065666026
0.98239	205.8220	0.71984	0.074607477
1.02218	219.0478	0.75958	0.084907678
1.06237	231.5507	0.80132	0.097255920
1.10218	243.0187	0.84405	0.111737914
1.14205	253.5626	0.88802	0.129273054
1.18264	263.3225	0.93368	0.150868505
1.23835	275.1766	0.99721	0.189418000